Unbalanced Multiphase Load-Flow Using a Positive-Sequence Load-Flow Program

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***Abstract*—Existing commercial load-flow packages have the capability of solving large balanced transmission systems using little computer memory and processing time. Their computation efficiency overcomes the small imprecision caused by the modeling simplifications and assumptions made in both transmission and distributions simulation packages. However, the interest for modeling more accurately unbalanced multiphase networks and solving complex integrated transmission-distribution systems has greatly increased among the power system specialists. The present work develops an unbalanced multiphase load-flow algorithm with the capability to model all components and network features found in power systems. The proposed algorithm allows the use of any existing positive-sequence load-flow solver as the main engine. The simulation results validated with EMTP-RV and CYMDIST prove that the proposed methodology has good numerical accuracy, robustness and efficiency.**

***Index Terms*—Electromagnetic couplings, multiphase systems, positive-sequence, transmission and distribution networks, unbalanced load-flow.**

I. INTRODUCTION

MOST load-flow studies in transmission systems are executed for balanced networks adopting a positive-sequence (or a single-phase) representation of lines, loads and all other devices. Positive-sequence load-flow assumptions such as perfect line transposition and balanced loads have shown to be quite accurate simplifications for representing transmission networks. However, there are cases in which a balanced representation is not accurate enough, as is often the case with distribution systems. There are also transmission cases where electromagnetic unbalance due to the non-transposition of phases are present and cannot be neglected. As a consequence, a more precise representation of all components found in electrical networks is not only desirable but needed. Many load-flow methodologies and numerical solutions have been proposed in the past years. Some of them have been implemented in commercial software packages, but few are able to solve the particular configurations often found in distribution systems. For example, explicit representation of neutral conductors and ground wires is not commonly available. Additionally, none of the existing algorithms (programs) has been implemented in a positive-sequence solver to obtain the solution of multiphase unbalanced systems.

The objective of the present work is to develop an algorithm that solves the multiphase load-flow problem for load-unbalanced systems in the presence of electromagnetically unbalanced lines using an existing positive-sequence load-flow solver.

The methodology presented in this paper relies on a positive-sequence representation of all the electromagnetically coupled (or uncoupled) multiphase components present in electrical networks such as generators, transmission lines, cables, transformers, shunt and series compensators, loads and voltage-controlled devices. The numerical solution is obtained with a conventional positive-sequence load-flow program, CYME's PSAF, based on the Newton algorithm of [1] with optimal ordering and elimination of the Jacobian matrix [2].

With the algorithm of this paper one is capable of analyzing the following features of actual power systems:

- tens of thousands of buses of varied voltage levels to cover simultaneously generation, transmission, distribution, industrial, and residential networks;
- any single- and three-phase transformer configuration (Yg-D, D-Yg, Yg-Yg, D-Yn, D-D, OpenD-OpenD, Yg-D mid-tap, etc.) including fixed or under load tap-changers;
- slack and PV controlled sources;
- meshed and radial configurations with any number of phases and connections;
- sections with any number of wires (transposed or not) such as: single-phase, single-phase plus neutral, single-phase plus neutral plus ground, two-phase, two-phase plus neutrals, two-phase plus neutrals plus ground, three-phase, three-phase plus any number of neutral conductors and ground wires, and so forth;
- balanced and unbalanced loads connected between phases, phase-to-ground, phase-to-neutral and phase-merging (loads connected between phases of two different branches or feeders). Loads can be modeled as constant power, constant impedance or constant current.

The EMTP-RV multiphase load-flow package has been used for validation purposes due to its well-known and proven performance. EMTP-RV uses its own computational method to solve the load-flow for unbalanced and multiphase networks in the phase-domain [3], and for that reason it is appropriate to validate the proposed methodology. In addition, the fast iterative solver used by CYMDIST [4] which is based on backward-forward

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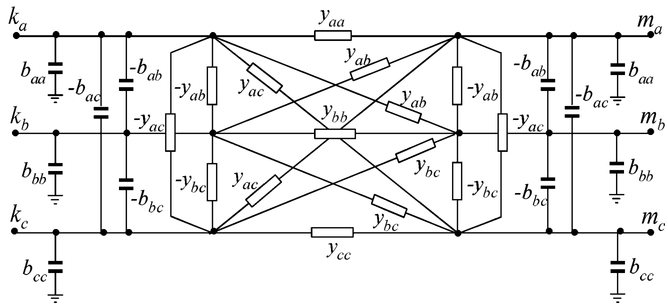

Fig. 1. Positive-sequence equivalent circuit of a three-phase mutually coupled line section.

sweeps method [5] is used to compare the accuracy and performance of the proposed methodology for a load unbalanced and continuously transposed large three-phase distribution system.

II. Multiphase Load-Flow Methodology

The methodology used in this paper relies on the models developed in [6] and enhanced in [7] to represent electromagnetically coupled components (lines, transformers, loads, etc.) with electromagnetically uncoupled positive-sequence impedances. The method produces no additional buses in the admittance matrix of a multiphase system, maintaining the same structure of the original network. This positive-sequence (or uncoupled) multiphase representation can be derived and understood in terms of elementary graph theory. Subsequently, the branch elements can be determined by inspection for the majority of the cases. The resulting uncoupled elements can be easily added to the admittance matrix of the network independently of the size of the system. For multiphase line and cable sections the inversion of their primitive impedance matrix, of order 3 for a three-phase system, is needed to get the admittance matrix to be entered into the final $\boldsymbol{Y_{Bus}}$ system matrix [8]. This process is done before the Newton iterations start and therefore it adds very little computer time.

Once the equivalent circuits for all the elements of the system have been obtained, we use a standard positive-sequence load-flow solver to obtain the accurate solution of a multiphase unbalanced system. This method is theoretically exact and numerically stable. It is proven to be computationally robust for transmission and distribution systems of arbitrary complexity.

A. Lines and Cables

For load-flow studies, a balanced three-phase line is typically represented only by its positive-sequence series impedance in addition to two shunt admittances (π model). In the proposed methodology, a three-phase line section is represented by 27 artificial lines, where 15 of them represent the impedance matrix and the other 12 the capacitive mutual couplings as is shown in the equivalent circuit representation of Fig. 1 [7], [8].

The bus admittance matrix $\boldsymbol{Y_{Bus\_A}}$ of the three-phase line circuit in Fig. 1 is derived by multiplying the inverse of the primitive impedance matrix $\boldsymbol{Z^{abc}}$ by its branch-bus incidence matrix $\boldsymbol{N}$, as follows:

$$\boldsymbol{Y_{Bus\_A}} = \boldsymbol{N}^T(\boldsymbol{Z^{abc}})^{-1}\boldsymbol{N} = \begin{bmatrix} \boldsymbol{Y^{abc}_{kk}} & -\boldsymbol{Y^{abc}_{km}} \\ -\boldsymbol{Y^{abc}_{mk}} & \boldsymbol{Y^{abc}_{mm}} \end{bmatrix} \quad (1)$$

where each sub-admittance matrix $\boldsymbol{Y^{abc}}$ in (1) has the following structure:

$$\boldsymbol{Y^{abc}_{kk,mm,km,mk}} = \begin{bmatrix} y_{aa} & y_{ab} & y_{ac} \\ y_{ab} & y_{bb} & y_{bc} \\ y_{ac} & y_{bc} & y_{cc} \end{bmatrix}. \quad (2)$$

The mutual capacitive elements represented by the matrix $\boldsymbol{B_{Bus\_C}}$ are determined as follows:

$$\boldsymbol{B_{Bus\_C}} = \begin{bmatrix} \boldsymbol{B^{abc}_{kk}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B^{abc}_{mm}} \end{bmatrix} \quad (3)$$

where each sub-admittance matrix $\boldsymbol{B^{abc}}$ has the following structure:

$$\boldsymbol{B^{abc}_{kk,mm}} = \begin{bmatrix} b_{aa} & b_{ab} & b_{ac} \\ b_{ab} & b_{bb} & b_{bc} \\ b_{ac} & b_{bc} & b_{cc} \end{bmatrix}. \quad (4)$$

Therefore, six additional series and six shunt capacitive elements need to be entered in the system admittance matrix. The mutual susceptances $-b_{ab}$, $-b_{bc}$ and $-b_{ac}$ in Fig. 1 should not add up to the sum of the diagonal elements of the system matrix. Therefore, we need to add the difference in a new capacitor to ground whose per phase values are given by

$$b_{ii} = b^{old}_{ii} + \sum_{j \neq i} b_{ij} \quad i, j = (a, b, c). \quad (5)$$

These shunt impedances need to be set in both sides of the transmission lines connecting buses, thus the final matrix for the line section in Fig. 1 has the following form:

$$\boldsymbol{Y_{Bus\_L}} = \begin{bmatrix} \boldsymbol{Y^{abc}_{kk}} + \boldsymbol{B^{abc}_{kk}} & -\boldsymbol{Y^{abc}_{km}} \\ -\boldsymbol{Y^{abc}_{mk}} & \boldsymbol{Y^{abc}_{mm}} + \boldsymbol{B^{abc}_{mm}} \end{bmatrix} \quad (6)$$

where the elements of submatrices $\boldsymbol{Y^{abc}}$ and $\boldsymbol{B^{abc}}$ in (2) and (4) correspond to those displayed in Fig 1.

The three-wire line model of Fig. 1 can represent a three-phase line, a two-phase line plus the explicit representation of the neutral conductor, or a single-phase line plus the explicit representation of the neutral conductor and ground wire.

Similar models to the one shown in Fig. 1 can be derived for the representation of a line section with any number of circuits and phases. For example, a two-wire model using 12 artificial lines can be used to represent a two-phase line, or a single-phase line plus the neutral conductor or ground wire. A four-wire three-phase line section, which can represent a three-phase line plus neutral conductor, a three-phase line plus ground or a two-phase line plus neutral wire and ground, has 48 artificial lines, where 28 of them represent the impedance matrix and the other 20 the admittance. The number of artificial lines for a general n-wire line is $(2n^2 - n)$ for impedance and $(n^2 + n)$ for admittance elements totalizing $3n^2$ artificial lines. The flexibility

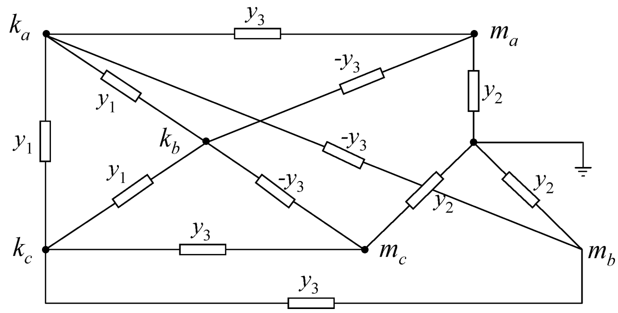

Fig. 2. Positive-sequence equivalent circuit of a three-phase delta-grounded wye transformer.

of this method makes it adequate for the analysis of distribution networks.

B. Transformers

The method to model transformers, developed in [9], computes the admittance matrix formed by the connection of single-phase transformer units. Models for all transformer connections can be derived with this methodology, including all unusual connections frequently found in distribution systems. Based on the assumption of single-phase units connected in three-phase systems, the transformer admittance matrices are developed following the same nodal approach as for the line model. Thus, the admittance matrix $\boldsymbol{Y_{Bus\_T}}$ of a three-phase delta to wye-grounded transformer is computed as follows:

$$Y_{Bus\_T} = \begin{bmatrix} \frac{2y_t}{3\alpha^2} & \frac{-y_t}{3\alpha^2} & \frac{-y_t}{3\alpha^2} & \frac{-y_t}{\sqrt{3}\alpha\beta} & 0 & \frac{y_t}{\sqrt{3}\alpha\beta} \\ \frac{-y_t}{3\alpha^2} & \frac{2y_t}{3\alpha^2} & \frac{-y_t}{3\alpha^2} & \frac{y_t}{\sqrt{3}\alpha\beta} & \frac{-y_t}{\sqrt{3}\alpha\beta} & 0 \\ \frac{-y_t}{3\alpha^2} & \frac{-y_t}{3\alpha^2} & \frac{2y_t}{3\alpha^2} & 0 & \frac{y_t}{\sqrt{3}\alpha\beta} & \frac{-y_t}{\sqrt{3}\alpha\beta} \\ \frac{-y_t}{\sqrt{3}\alpha\beta} & \frac{y_t}{\sqrt{3}\alpha\beta} & 0 & \frac{y_t}{\beta^2} & 0 & 0 \\ 0 & \frac{-y_t}{\sqrt{3}\alpha\beta} & \frac{y_t}{\sqrt{3}\alpha\beta} & 0 & \frac{y_t}{\beta^2} & 0 \\ \frac{y_t}{\sqrt{3}\alpha\beta} & 0 & \frac{-y_t}{\sqrt{3}\alpha\beta} & 0 & 0 & \frac{y_t}{\beta^2} \end{bmatrix} \tag{7}$$

where y_t is the per unit leakage admittance of the single-phase transformers while α, and β represent the off-nominal tap at the primary and the secondary sides of the transformer respectively. The three-phase equivalent circuit with its artificial lines can be obtained by inspection of the admittance matrix (7), and the resulting circuit is shown in Fig. 2. The values for y_1, y_2 and y_3 are given in [9].

Other transformer connections, whose models have been developed in [10], are the open wye-open delta and open delta-open delta transformers. These connections permit to supply, for instance, a two-phase plus neutral wire network from a three-phase system on the primary side, allowing feeding two-phase or single-phase to neutral loads, which are standard in distribution systems.

In general, distribution loads are unbalanced, involving combinations of single-phase and three-phase loads in the same feeder. Consequently, three-phase four-wire distribution transformers banks with grounded mid-tap on the secondary side are widespread in distribution networks [11]. These transformers are composed of either one single-phase transformer with three secondary terminals, two phase terminals and one neutral terminal, or one or two single-phase transformers with only two phase terminals. Three-phase four terminal distribution transformers allow the operation under unbalanced situations and feed both single-phase and three-phase loads.

C. Generators

In traditional positive-sequence (balanced) load-flow studies, generators are represented as fixed voltage sources (slack), PV sources or as negative PQ loads neglecting their internal impedances. However, since internal impedances are coupled in actual three-phase generators, the terminal voltages (both magnitude and angle) of a PV generator can be unbalanced. It is also possible to have different power generated per phase. In the case of a PQ generator model, both P and Q of each phase could be different. Nevertheless, the internal voltage of the generator will always be balanced, same magnitude with the phases 120° apart.

In general a three-phase generator is represented by its Thevenin equivalent equation as follows:

$$\boldsymbol{E_{abc}} - \boldsymbol{Y_{abc}^{-1}}\boldsymbol{I_{abc}} = \boldsymbol{V_{abc}} \tag{8}$$

where $\boldsymbol{E_{abc}}$ and $\boldsymbol{V_{abc}}$ are the internal and terminal voltage vectors and $\boldsymbol{Y_{abc}}$ the internal admittance matrix which is computed from the (measured) sequence impedances $\boldsymbol{Z_{012}}$ using the Fortescue (symmetrical components) transformation matrix $\boldsymbol{T}$ as

$$\boldsymbol{Y_{abc}} = \boldsymbol{T}\boldsymbol{Z_{012}^{-1}}\boldsymbol{T^{-1}}. \tag{9}$$

The values of the vectors $\boldsymbol{V_{abc}}$ and $\boldsymbol{I_{abc}}$ depend on the bus type. For a three-phase slack generator bus, the terminal voltage vector is known and the active and reactive powers are determined by the system load-flow. For a PV generator bus, the voltage magnitude of phase a and the active power are known, while the voltage angle and reactive power are the unknown variables.

Since matrix $\boldsymbol{Y_{abc}}$ is non-symmetric, it cannot be represented by an equivalent electric circuit to be entered directly in a symmetric positive-sequence solver. However, the generator terminal voltages can be computed accurately with the iterative method described next. The first step is to start with an initial guess and solve the load-flow as usual. Next, the controlled voltage is taken as the terminal voltage magnitude of phase a. Finally, (8) is used to compute the magnitude and angle of the internal voltage E_a. Because the internal voltages must be balanced, the internal voltages of phases b and c are forced to be equal to $|E_a|$ in module with the angles shifted by $-120°$ and 120° respectively. With the new values the process is restarted again until convergence is achieved.

The above iterative methodology, which is theoretically exact, is recommended by the authors for the representation of synchronous generators in three-phase load-flow problems. Alternatively, even though it is well-known that the positive-sequence reactance of synchronous generators is different from its subtransient reactance, the latter can be used to represent the impedance of a generator for load-flow studies when there is phase unbalance as stated by Chen in [9].

A test was made to gauge the effect of assuming that both the positive and negative-sequence impedances are equal to the

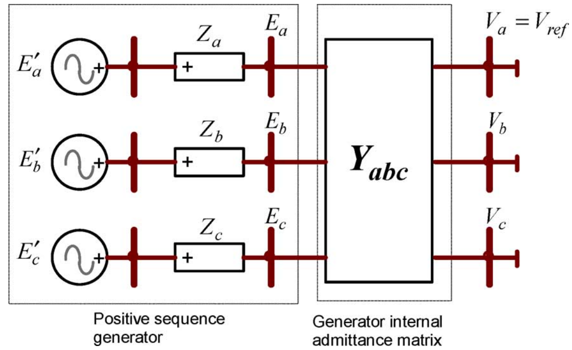

Fig. 3. Positive-sequence equivalent model of a three-phase generator.

subtransient reactance. The largest errors found using this approximation for a generator feeding a current load unbalance (I_2/I_1) of 10% were 0.001% and 0.03% for the magnitude and angle respectively at the generator terminals. Note that under the above conditions the internal impedance matrix of the generator becomes symmetric.

Consequently, a three-phase synchronous generator can be modeled using three positive-sequence generators whose internal impedances Z_a, Z_b and Z_c are set to zero. The impedance is then externally represented with acceptable accuracy by an unbalanced but symmetric matrix $\boldsymbol{Y_{abc}}$. The generator model is shown in Fig. 3. There is a remote control relation between the internal voltage E_a and reference voltage V_{ref}.

D. Loads

Typical load models known as constant current, constant impedance and constant power can be included in the methodology of this paper.

Constant power loads can be easily represented in a positive-sequence load-flow analysis by defining explicitly the values of P and Q as a balanced load connected to an infinite ground. However, in distribution systems, loads are often found connected to neutral or between phases. Thus, the positive-sequence representation does not work properly.

An iterative voltage-dependent impedance-based method is proposed here. A single-phase load is represented with an impedance that changes with voltage. Three-phase unbalanced loads are represented by three wye or delta connected single-phase impedances.

Let us define S_S, V_S, I_S and Z_S as the specified nominal complex power, voltage, current and impedance of a single-phase load. Thus for a constant power load, S_S is given by

$$S_S = \frac{|V_S|^2}{Z_S^*} = P_S + jQ_S. \tag{10}$$

Solving (10) for Z_S we get

$$Z_S = \left(\frac{P_S + jQ_S}{P_S^2 + Q_S^2}\right)|V_S|^2. \tag{11}$$

Defining α as

$$\alpha = \frac{P_S + jQ_S}{P_S^2 + Q_S^2} \tag{12}$$

we find an impedance that is a quadratic function of the voltage magnitude. Thus, the corrected impedance Z_n^{corr} at iteration n which will be used in iteration $n+1$ is computed from

$$Z_n^{corr} = \alpha|V_n|^2. \tag{13}$$

Equation (13) can be used iteratively, in the Gauss–Seidel sense, with the load-flow equations to obtain the accurate solution in the presence of constant power (PQ) loads.

Another formulation, compatible with the Newton method adopted in this paper, can be obtained by linearizing (13) for small variations of voltage ΔV_n between iterations. Then the corrected impedance Z_n^{corr} to be used in the $n+1$ iteration is obtained by extrapolating over the tangent

$$Z_n^{corr} = Z_n + \Delta Z_n. \tag{14}$$

The correction factor is computed with the first derivative as

$$\Delta Z_n = \frac{\partial Z_n}{\partial V_n}\Delta V_n. \tag{15}$$

Substituting (13) in (15) and taking the derivative we have

$$\Delta Z_n = 2\alpha|V_n|\Delta V_n. \tag{16}$$

Note that Z_n and ΔZ_n are functions of the applied voltage and the voltage mismatch according to (13) and (16) respectively. Replacing these equations in (14), we get the expression for the corrected impedance value Z_n^{corr} as

$$Z_n^{corr} \approx \alpha\left[V_n^2 + 2|V_n|\Delta V_n\right]. \tag{17}$$

The same approach is applied for the constant current load model, but in this case the dependence of the impedance with the voltage is linear. Thus, the corrected impedance value Z_n^{corr} at the iteration n is

$$Z_n^{corr} = \beta V_n + \beta\Delta V_n \tag{18}$$

with

$$\beta = \frac{1}{I_S} = \frac{V_S^*}{P_S - jQ_S}. \tag{19}$$

Using (17) and (18) we are able to model all kinds of loads found in distribution systems. Delta- or wye-connected, single-, two-, and three-phase loads being either balanced or unbalanced can be represented with this iterative method. The number of iterations required for the proposed load model usually does not exceed four. Since the Newton method itself takes from three to five iterations to converge, the impedance correction process may not increase the number of iterations.

E. Neutral-Wire and Ground

Conventional positive-sequence load-flow software does not include explicit representation of neither the neutral conductors nor the ground wires.

The neutral and ground wires are usually considered implicitly by applying the Kron-reduction and Carson's equations [5]

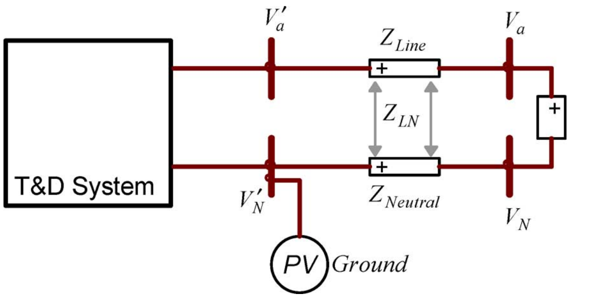

Fig. 4. Explicit zero-voltage reference bus in a positive-sequence load-flow representation.

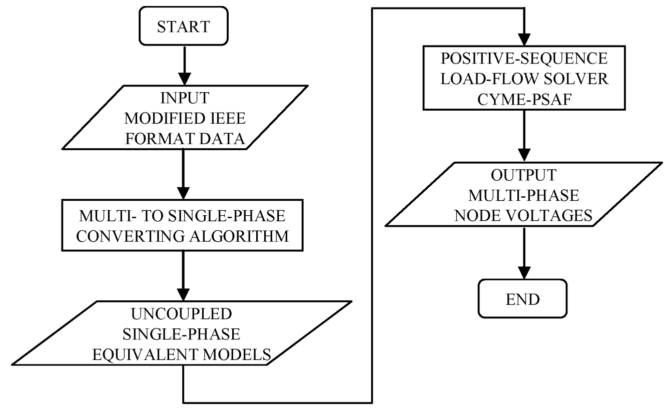

Fig. 5. Multiphase load-flow algorithm flowchart.

to represent the neutral-conductor and ground respectively. In the proposed methodology, both neutral conductors and ground wires can be represented explicitly as simple positive-sequence lines [12].

The infinite ground or zero reference voltage is also explicitly included in the methodology of this paper. For that purpose a zero-voltage bus is proposed. In distribution systems, zero-voltage reference or ground is mainly found in single-phase to neutral loads, grounded wye-connected transformers and generators. The most effective zero-voltage reference bus representation is obtained by adding a PV bus at each required ground node whose voltage and power are set to zero as shown in Fig. 4. By adding a zero PV reference constraint it is possible to represent explicitly the ground in any system using a positive-sequence load-flow solver. Ground impedances can be also added between the neutral and the zero-reference node if needed.

F. Initial Conditions

A favorable starting approximation is frequently necessary for the successful convergence of large systems when Newton's method is used. The flat voltage start, where voltage magnitudes are set equal to their scheduled (or nominal) values and angles equal to the slack node voltage angle, is usually sufficient. Thus, for a three-phase program the angles at each node is set equal to the angle of the slack reference bus, which is usually $0°$, $-120°$, and $120°$ for phases a, b and c respectively. Phase-shift due to transformer connections causes leads or lags and the establishment of power flow.

III. Three-Phase Load-Flow Algorithm

The algorithm developed is composed of two main modules. The first module has the function of converting a three-phase magnetically-coupled network into an uncoupled positive-sequence equivalent network. The second module and main engine is the existing balanced positive-sequence load-flow solver CYME's PSAF. The main advantage of the proposed methodology is its capability to simulate multiphase networks using any positive-sequence solver readily available. A general description of the three-phase algorithm is depicted in the flowchart of Fig. 5.

For entering data, a multiphase format was developed which is a modification of the standard IEEE common data for the exchange of solved load-flow data [13]. The multi- to single-phase conversion algorithm has the purpose of transforming a multiphase system into an equivalent uncoupled positive-sequence system. As a result of the conversion process, an output data file is generated in a suitable format, which is read by the positive-sequence load-flow solver. The positive-sequence load-flow algorithm as mentioned above uses the Newton's methodology [1] with the optimal sparse matrices ordering proposed in [2].

The output data format includes the magnitude and the phase angle of the voltage at each node of the system. The node voltage data are used to validate the proposed methodology against the results obtained from the EMTP-RV load-flow package and CYMDIST. The latter was used not only to validate systems including ideally transposed lines and excluding explicit neutral-wire representation, but also to establish a performance benchmark.

IV. Test Cases and Results Validation

Three test systems were used to assess the accuracy, performance and robustness of the proposed methodology. The first test case corresponds to the IEEE 34-bus unbalanced distribution system [14]. The objective of this test case is to validate the accuracy of the method when solving an unbalanced and non-transposed radial distribution network against the software EMTP-RV.

The second test case corresponds to a 40-bus system developed with the purpose of validating most of the devices and complexities found in distribution and transmission systems such as: generators, transformers with different connections, meshed and radial networks, coupled and uncoupled lines, different load representations, single-, double-, three- and four-wire lines, three-phase, phase-to-phase, and phase-to-neutral loads, and fixed capacitor banks. Many of these characteristics are hardly ever modeled in either conventional positive-sequence load-flow or in fast iterative radial distribution applications.

The third test case corresponds to a very large and highly meshed unbalanced network with more than 2600 nodes and ideally transposed lines. This test case is used to evaluate both robustness and performance of the developed algorithm in comparison with the EMTP-RV and the fast iterative software implemented in CYMDIST. This case was modeled using continuously transposed lines since CYMDIST, although efficient, does not allow at present to model non-transposed networks. All three cases are available to readers upon request.

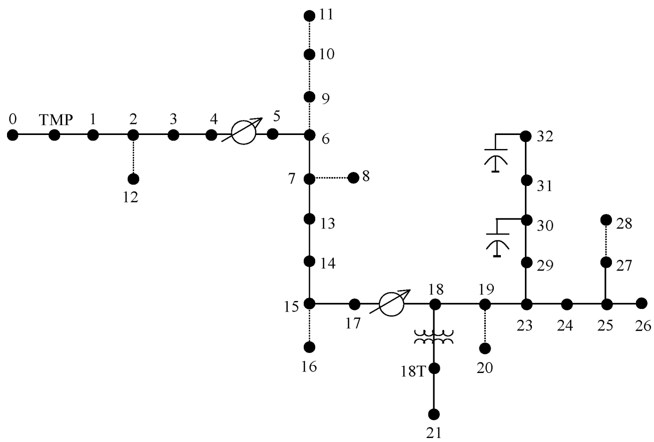

Fig. 6. IEEE 34-bus distribution system; dotted lines are single-phase.

TABLE II
IEEE 34-BUS TEST SYSTEM—VOLTAGE ANGLE RESULTS

Bus	Voltage Angle [deg]					
	EMTP-RV Phase A	Method Phase A	EMTP-RV Phase B	Method Phase B	EMTP-RV Phase C	Method Phase C
0	0.0	0.0	-120.0	-120.0	120.0	120.0
3	-1.3	-1.3	-121.1	-121.0	119.3	119.3
4	-1.8	-1.8	-121.6	-121.4	118.9	119.1
11	-2.5	-2.5				
16			-122.2	-122.0		
17	-3.1	-3.1	-122.7	-122.4	117.4	117.6
20			-123.0	-122.7		
21	-6.9	-6.9	-126.2	-126.0	112.8	113.0
26	-3.5	-3.5	-123.0	-122.7	116.9	117.2
28			-123.0	-122.7		
29	-3.5	-3.5	-123.0	-122.8	116.9	117.1
32	-3.6	-3.5	-123.1	-122.8	116.8	117.1

TABLE I
IEEE 34-BUS TEST SYSTEM—VOLTAGE MAGNITUDE RESULTS

Bus	Voltage Magnitude [pu]					
	EMTP-RV Phase A	Method Phase A	EMTP-RV Phase B	Method Phase B	EMTP-RV Phase C	Method Phase C
0	1.050	1.050	1.050	1.050	1.050	1.050
3	0.957	0.955	0.976	0.976	0.949	0.948
4	0.919	0.918	0.946	0.946	0.908	0.907
11	0.919	0.920				
16			0.929	0.928		
17	0.876	0.877	0.894	0.893	0.830	0.830
20			0.920	0.919		
21	0.753	0.753	0.779	0.778	0.674	0.675
26	0.900	0.901	0.916	0.915	0.847	0.848
28			0.916	0.915		
29	0.901	0.901	0.917	0.915	0.848	0.848
32	0.901	0.901	0.916	0.915	0.848	0.848

A. *IEEE 34-Bus Distribution System Test Case*

The IEEE 34-bus unbalanced radial distribution feeder is shown in Fig. 6. The data, loads and line parameters, were obtained from [14]. This IEEE 34-bus test case corresponds to a radial distribution feeder with unbalanced loads and non-transposed lines. It contains both single-to-ground and three-phase constant power loads. Tables I and II show the calculated voltage magnitudes and angles for a list of selected nodes having the largest mismatches. It can be observed that the results are quite similar for both solvers. For a tolerance of 10^{-6} MVA, the largest errors found are 0.21% and 0.24% for the voltage magnitude and angle respectively. Grey cells in Tables I and II refer to the nonexistent buses of single-phase sections.

B. *The 40-Bus Transmission and Distribution Test Case*

Most commercially available load-flow programs are able to model and simulate balanced transmission or unbalanced distribution networks. A few can integrate and simulate both systems at the same time. Large R/X ratios, line transposition, load unbalancing, highly meshed distribution networks, explicit neutral wire representation, phase-to-phase, phase-to-neutral and phase-merged loads are among the typical features found in real networks and which are hardily ever modeled by commercial load-flow software. EMTP-RV load-flow package is one exception and that is the reason for choosing it as the validation tool.

With the methodology proposed here any positive-sequence load-flow can be used to solve the complex system presented in this second test. Fig. 7 shows the 40-bus test case developed for validation purposes. The calculated magnitudes and angles for selected buses are shown in Tables III and IV. It can be observed that the largest errors found are 0.55% and 0.53% for the voltage magnitude and angle respectively.

The small round-off errors found in some nodes is mainly due to the numerical precision in the data converting algorithms. Other smaller source for error is the use of the symmetric approximation for the internal impedance of the generators.

Note that the 40-bus test system has a very large voltage ratio, defined as the quotient between the maximum and the minimum nominal voltages in the system. This affects the p.u. values of the impedances and loads. A typical transmission or distribution system has a voltage ratio that varies from 10 to 30. However, composed transmission-distribution systems, such as our example, have voltage ratios from 500 to 3000.

The main difference between EMTP-RV and the methodology presented in this work is that EMTP-RV is a tool conceived as a multiphase load-flow in the phase domain and the proposed methodology can be implemented in any existing positive-sequence program.

C. *The 2600-Bus Distribution Test Case*

A third test involving a large system with more than 2600 buses was also simulated in order to compare the performance and the robustness of the proposed methodology against EMTP-RV and CYMDIST. The performance has been tested on a Pentium Duo Core, 3.2-GHz speed processor with 2-GB RAM computer. The simulation time and number of iterations for each solver are presented in Table V.

The total simulation time presented in Table V includes the data reading time, which is not minor for large networks. Considering this and taking into account that the proposed methodology has not been optimized in terms of programming, the simulation time seems to be quite reasonable for a large three-phase system.

The fast iterative sweep solver presents the smallest simulation time, but the largest number of iterations due to the large number of loops in the system. EMTP-RV, which is optimized

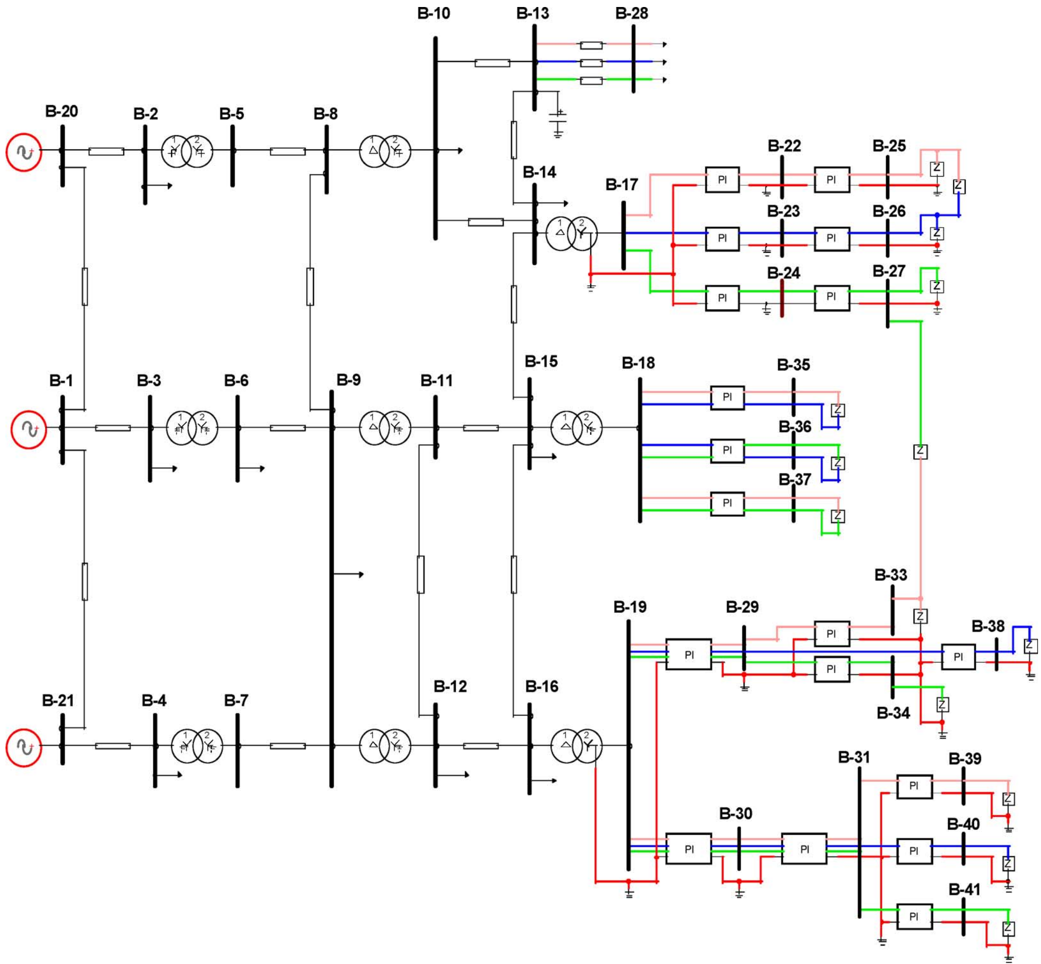

Fig. 7. The 40-bus transmission and distribution test system.

TABLE III
THE 40-BUS TEST SYSTEM—VOLTAGE MAGNITUDE RESULTS

Bus	Voltage Magnitude [pu]					
	EMTP-RV Phase A	Method Phase A	EMTP-RV Phase B	Method Phase B	EMTP-RV Phase C	Method Phase C
1	1.000	1.000	1.000	1.000	1.000	1.000
10	0.983	0.983	0.984	0.984	0.979	0.979
24					0.943	0.945
25	0.923	0.928				
26			0.885	0.887		
27					0.913	0.918
33	0.903	0.905				
36			0.960	0.959	0.944	0.944
39	0.947	0.950				
41					0.935	0.938

TABLE IV
THE 40-BUS TEST SYSTEM—VOLTAGE ANGLE RESULTS

Bus	Voltage Angle [deg]					
	EMTP-RV Phase A	Method Phase A	EMTP-RV Phase B	Method Phase B	EMTP-RV Phase C	Method Phase C
1	0.0	0.0	-120.0	-120.0	120.0	120.0
10	28.4	28.4	-91.8	-91.8	148.0	148.0
24					175.6	175.5
25	53.4	53.3				
26			-62.9	-63.2		
27					173.1	172.8
33	56.5	56.2				
36			-63.8	-63.8	178.2	178.2
39	56.2	56.1				
41					175.9	175.7

to find the load-flow solution and move quickly into its time-domain initialization, takes a slightly shorter total simulation time than the proposed methodology. However, the performance of the latter can be improved by eliminating some overhead in data input reading and by further optimization in the computational process. The longer solution time exhibited by the EMTP-RV is caused by the pivoting and ordering operations performed at each iteration and the use of a non-symmetric system matrix. On the other hand, the methodology of this paper takes longer time for the reading process because each line is represented by a large number of artificial lines. The important fact is that the number of iterations, which accounts for the iterations required for the impedance correction method used to model constant power loads, is low and almost the same in both solvers. The largest voltage magnitude and angle mismatches are smaller than 0.1% for this test case.

V. CONCLUSIONS

The present work has involved the development of a methodology for solving multiphase load-flow problems using any existing positive-sequence solver. The solution methodology heavily relies on the existing equivalent positive-sequence (or

TABLE V
THE 2600-BUS DISTRIBUTION TEST CASE—PERFORMANCE RESULTS

Solver	Simulation Time [s]			Number of iterations
	Reading	Solution	Total	-
CYMDIST	1.5	3.5	5.0	77
EMTP-RV	0.6	11.6	12.2	3
New Method	6.7	6.4	13.1	4

uncoupled) models for multiphase mutually-coupled system components.

The integration of both transmission and distribution systems in one simulation as well as the ability of accurately model all typical devices found in real transmission and distribution systems are the major contributions of this research work. The main advantages of the proposed methodology can be summarized as follows:

- the capability of solving multiphase systems with their corresponding electromagnetic coupling, which is very attractive for distribution networks applications involving explicit neutral conductor and ground wire representation;
- explicit representation of different types of loads such as single-, double- or three-phase. Loads can be connected in delta or wye and be balanced or unbalanced;
- transposed and non-transposed lines can be modeled with this methodology due to its ability of representing electromagnetic couplings between phases as well as parallel or nearby lines. Both radial and highly meshed networks can be solved thanks to the matrix formulation adopted and robust convergence properties;
- all kinds of transformers connection can be represented with this methodology including many special connections found in distribution networks such as open-wye, open-delta, and mid-tap transformers;
- the capability of modeling and simulating large composed transmission and distribution systems, and complex configuration such as phase-merged loads, which are loads connected between phases of different branches or feeders.

The simulation results have shown to be very accurate and the number of iterations of the developed algorithm is low showing the high robustness of the presented methodology. The performance test using the 2600-bus case has shown that the simulation time, including data reading, reached by the developed algorithm is close to the time obtained with EMTP-RV. Future work will be aimed to improve and optimize the algorithm to reduce the simulation time.

REFERENCES

[1] W. F. Tinney and C. E. Hart, "Power flow solution by Newton's method," *IEEE Trans. Power App. Syst.*, vol. PAS-86, no. 11, pp. 1449–1460, Nov. 1967.

[2] W. F. Tinney and J. W. Walker, "Direct solutions of sparse network equations by optimally ordered triangular factorization," *Proc. IEEE*, vol. 55, no. 11, pp. 1801–1809, Nov. 1967.

[3] J. Mahseredjian, S. Dennetière, L. Dubé, B. Khodabakhchian, and L. Gérin-Lajoie, "On a new approach for the simulation of transients in power systems," *Elect. Power Syst. Res.*, vol. 77, no. 11, pp. 1514–1520, Sep. 2007.

[4] *"CYMDIST Version 4.7 Reference Manual,"* CYME International T&D, Inc. [Online]. Available: http://www.cyme.com.

[5] W. Kersting, *Distribution Systems Modeling and Analysis*. Boca Raton, FL: CRC, 2002.

[6] F. L. Alvarado, "Formation of Y-node using the primitive Y-node concept," *IEEE Trans. Power App. Syst.*, vol. PAS-101, no. 12, pp. 4563–4572, Dec. 1982.

[7] T. H. Chen, M. S. Chen, K. J. Hwang, P. Kotas, and E. A. Chebli, "Distribution system power flow analysis: A rigid approach," *IEEE Trans. Power Del.*, vol. 6, no. 3, pp. 1146–1152, Jul. 1991.

[8] H. A. Smolleck and R. R. Shoults, "A straightforward method for incorporating mutually-coupled circuits into the bus admittance matrix using the concept of artificial branches," *IEEE Trans. Power Syst.*, vol. 5, no. 2, pp. 486–491, May 1990.

[9] T. H. Chen, M. S. Chen, T. Inoue, P. Kotas, and E. A. Chebli, "Three-phase co-generator and transformer models for distribution system analysis," *IEEE Trans. Power Del.*, vol. 6, no. 4, pp. 1671–1681, Oct. 1991.

[10] T. H. Chen and J. D. Chang, "Open wye-open delta and open delta-open delta transformer models for rigorous distribution system analysis," *Proc. Inst. Elect. Eng., Gen., Transm., Distrib.*, vol. 139, no. 3, pp. 227–234, May 1992.

[11] T. H. Chen and W. C. Yang, "Modeling and analysis of three-phase four-wire distribution transformers with mid-tap on the secondary side," in *Proc. Int. Conf. Energy Management and Power Delivery*, Mar. 1998, vol. 2, pp. 723–727.

[12] R. M. Ciric, A. P. Feltrin, and L. F. Ochoa, "Power flow in four-wire distribution networks-general approach," *IEEE Trans. Power Syst.*, vol. 18, no. 4, pp. 1283–1290, Nov. 2003.

[13] IEEE Working Group, "Common data format for the exchange of solved load flow data," *Trans. Power App. Syst.*, vol. PAS-92, no. 6, pp. 1916–1925, Nov./Dec. 1973.

[14] W. H. Kersting, "Radial distribution test feeders," *IEEE Trans. Power Syst.*, vol. 6, no. 3, pp. 975–985, Aug. 1991.

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